PH-107: Introduction to Quantum Mechanics
Tutorial Sheet 11

* marked problems will be solved in the tutorial class (D3-D4: Wednesday, D1-D2: Saturday)

Statistical Mechanics:

1. A national powerball lottery uses two sets of balls. The first set consists of 59 sequentially numbered balls and the second set consists of 35 sequentially numbered balls. Assume equal probability of choosing any ball and that all the balls are differently numbered. Five balls are chosen without replacement from the set of 59. Then one ball is chosen from the set of 35. Calculate the number of ways these six balls can be chosen (and thus your probability of winning the grand powerball prize).

2. Suppose we have 20 coins and we flip all of them together.

   (a) Considering all the coins to be independent of each other, how many possible outcomes (no. of microstates) do you expect with such a flipping?

   (b) How many ways are there for obtaining 12 heads and 8 tails?

   (c) What is the probability of obtaining 12 heads and 8 tails regardless of the order? They are called macrostates.

3. Three indistinguishable particles (say electrons) are to be arranged in three different energy levels of energy 0, E and $2E$, with respective degeneracies (ignore spin degeneracy) 2, 10 and 20. The total energy available is $3E$. What are the possible distributions and what are their probabilities?

4. * Consider a particle confined to a 3D harmonic oscillator potential, $V(x, y, z) = \frac{1}{2}m\omega^2(x^2 + y^2 + 4z^2)$

   (a) Calculate the ground state energy of the particle.

   (b) What is the degeneracy of the state with energy, $E = 7\hbar\omega$?

5. A certain thermodynamic system has non-degenerate energy levels, with energies 0, E, $3E$, $5E$ and $9E$. Suppose that there are four particles, with total energy $U = 9E$. Identify the possible distribution of particles and evaluate their microstates when (a) the particles are distinguishable, (b) the particles are identical bosons and (c) the particles are identical fermions.

6. In how many ways three electrons can occupy ten states (include spin degeneracy)? Is the number same as the way in which three persons can occupy ten chairs in a room? State the reason. In case the number is different, find the other number also.

7. The energy of a particle in a 3-D cubical box is given by

$$E_{n_x,n_y,n_z} = \frac{\pi^2\hbar^2}{2mL^2}\left(n_x^2 + n_y^2 + n_z^2\right)$$

If these levels are going to be occupied by electrons, write the energy values corresponding to the five lowest levels, taking into account the spin degeneracy. If three electrons occupy these states, find out the possible distributions which would yield a total energy of $18\pi^2\hbar^2/2mL^2$. Also find out the probability for each distributions.

A system has one state with energy 0, four states with energy $2E$ and eight states with energy $3E$. Six electrons are to be distributed among these states such that their total energy is $12E$. Consider a configuration (j, m, n) in which j electrons are in 0 energy state, m electrons are in $2E$ energy state and n electrons are in $3E$ state.

(a) Calculate the total number of microstates for the configuration (1,3,2).

(b) Find the ratio of probability of occurrence of a configuration (2,0,4) to that of a configuration (1,3,2)

8. * Consider a system of five particles trapped in a 1-D harmonic oscillator potential.

(a) What are the microstates of the ground state of this system for classical particles, identical Bosons and identical spin half Fermions.

(b) Suppose that the system is excited and has one unit of energy ($\hbar\omega$) above the corresponding ground state energy in each of the three cases. Calculate the number of microstates for each of the three cases.

(c) Suppose that the temperature of this system is low, so that the total energy is low (but above the ground state), describe in a couple of sentences, the difference in the behavior of the system of identical bosons from that of the system of classical particles.