PH-107: Introduction to Quantum Mechanics
Tutorial Sheet 9

* marked problems will be solved in the tutorial class (D3-D4: Wednesday, D1-D2: Saturday)

Scattering problems:

1. * A potential barrier is defined by $V = 0$ for $x < 0$ and $V = V_0$ for $x > 0$. Particles with energy E ($< V_0$) approaches the barrier from left.

   (a) Find the value of $x = x_0$ ($x_0 > 0$), for which the probability density is $1/e$ times the probability density at $x = 0$.

   (b) Take the maximum allowed uncertainty Δx for the particle to be localized in the classically forbidden region as x_0. Find the uncertainty this would cause in the energy of the particle. Can then one be sure that its energy E is less than V_0.

2. Consider a potential

$$\begin{aligned} V(x) &= 0 \ \text{ for } x < 0, \\ &= -V_0 \text{ for } x > 0 \end{aligned}$$

   Consider a beam of non-relativistic particles of energy $E > 0$ coming from $x \to -\infty$ and being incident on the potential. Calculate the reflection and transmission coefficients.

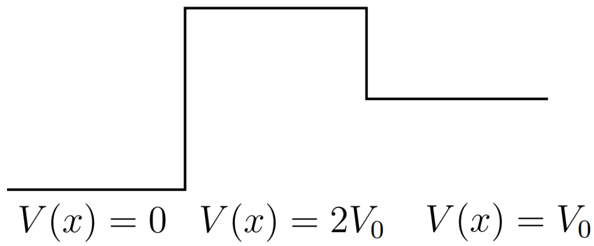
3. A potential barrier is defined by $V = 0$ eV for $x < 0$ and $V = 7$ eV for $x > 0$. A beam of electrons with energy 3 eV collides with this barrier from left. Find the value of x for which the probability of detecting the electron will be half the probability of detecting it at $x = 0$.

4. * A beam of particles of energy E and de Broglie wavelength λ, traveling along the positive x-axis in a potential free region, encounters a one-dimensional potential barrier of height $V = E$ and width L.

   (a) Obtain an expression for the transmission coefficient.

   (b) Find the value of L (in terms of λ) for which the reflection coefficient will be half.

5. A beam of particles of energy $E < V_0$ is incident on a barrier (see figure below) of height $V = 2V_0$. It is claimed that the solution is $\psi_I = A\exp(-k_1 x)$ for region I ($0 < x < L$) and $\psi_{II} = B\exp(-k_2 x)$ for region II ($x > L$), where $k_1 = \sqrt{\frac{2m(2V_0-E)}{\hbar^2}}$ and $k_2 = \sqrt{\frac{2m(V_0-E)}{\hbar^2}}$. Is this claim correct? Justify your answer.

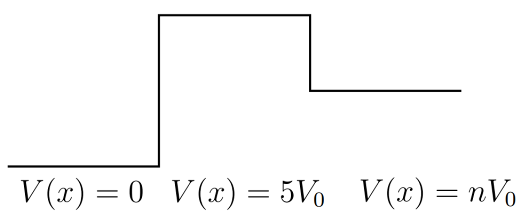


6. * A beam of particles of mass m and energy $9V_0$ (V_0 is a positive constant with the dimension of energy) is incident from left on a barrier, as shown in figure below. $V = 0$ for $x < 0$, $V = 5V_0$ for $x \leq d$ and $V = nV_0$ for $x > d$. Here n is a number, positive or negative and $d = \pi h/\sqrt{8mV_0}$. It is found that the transmission coefficient from $x < 0$ region to $x > d$ region is 0.75.

   (a) Find n. Are there more than one possible values for n?

   (b) Find the un-normalized wave function in all the regions in terms of the amplitude of the incident wave for each possible value of n.

   (c) Is there a phase change between the incident and the reflected beam at $x = 0$? If yes, determine the phase change for each possible value of n. Give your answers by explaining all the steps and clearly writing the boundary conditions used



7. A scanning tunneling microscope (STM) can be approximated as an electron tunneling into a step potential [$V(x) = 0$ for $x \leq 0$, $V(x) = V_0$ for $x > 0$]. The tunneling current (or probability) in an STM reduces exponentially as a function of the distance from the sample. Considering only a single electron-electron interaction, an applied voltage of 5 V and the sample work function of 7 eV, calculate the amplification in the tunneling current if the separation is reduced from 2 atoms to 1 atom thickness (take approximate size of an atom to be 3 Å).