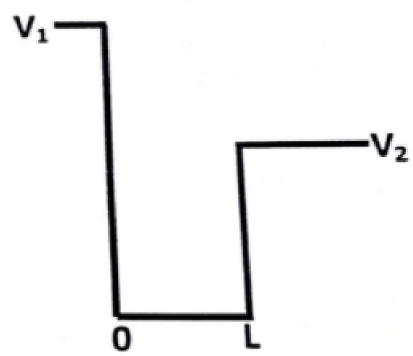
PH-107: Introduction to Quantum Mechanics
Tutorial Sheet 8

* marked problems will be solved in the Wednesday tutorial class

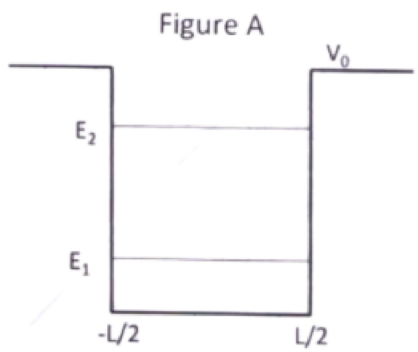
Particle in a Finite Box:

1. * Consider the asymmetric finite potential well of width L, with a barrier V_1 on one side and a barrier V_2 on the other side. Obtain the energy quantization condition for the bound states in such a well. From this condition derive the energy quantization conditions for a semi-infinite potential well (when $V_1 \to \infty$ and V_2 is finite).



2. Consider a particle of mass 'm' trapped in a finite square box of length 'a' with barrier height equals to V_0. Find the number of bound states and the corresponding energies for the finite square well potential if $\sqrt{ma^2V_0/\left(2\hbar^2\right)} = 1$).

3. An electron is trapped in a 1-dimensional symmetric potential well of height V_0 and width L (Figure A). The energy of electron is E, its wave number inside the well is k, and the magnitude of the wave number outside the well is α.



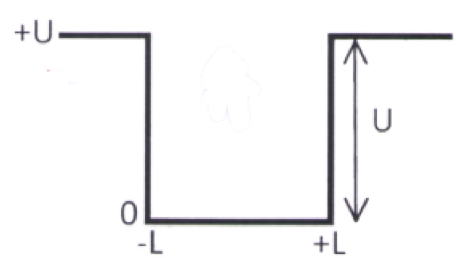
(a) Derive the energy quantization conditions, in terms of k, α and L, for the symmetric and anti-symmetric bound states.
(b) When the width of the well is L = 0.2 nm, it is found that the ground state energy $E_1 = 4.45$eV, and the first excited state energy $E_2 = 15.88$eV. Calculate V_0.
(c) Calculate the penetration depth for the ground state.
(d) If the width of the potential well is doubled to 2 L keeping V_0 the same, estimate the change in the ground state energy.
(e) Consider the potential which is generated from Figure A by setting $V = \infty$ at $L = 0$. What is the energy of the ground state in this case?
(f) How many bound states are possible in this case?

4. *Consider a particle of mass m in a potential given by

$$\begin{aligned} V(x) &= 0 \text{ for } |x| < L/2, \\ &= V_0 \text{ for } L/2 < |x| < L \\ &= \infty \text{ for } |x| \geq L \end{aligned}$$

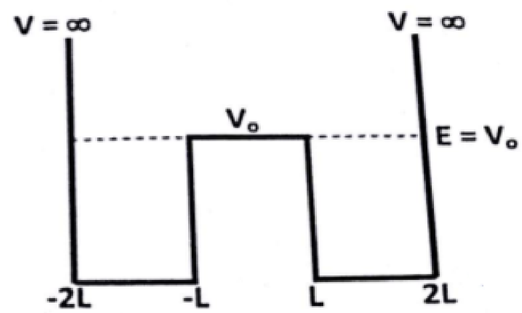
(a) Sketch the potential and the qualitative nature of the ground-state wave-function (without solving the Schrodinger equation). Mention the functional form of the wave function in each region.
(b) An electron is trapped in a symmetric finite potential of depth $V_0 = 1000$eV and width $L = 1$. What is approximate energy of the ground state?

5. A particle with energy E is bound in a finite square well potential with height U and width $2L$ (as shown in the figure below)



(a) Consider the case $E < U$, obtain the energy quantization condition for the symmetric wave functions in terms of K and α, where $K = \sqrt{2mE/\hbar^2}$ and $\alpha = \sqrt{2m(U-E)/\hbar^2}$
(b) Apply this result to an electron trapped at a defect site in a crystal. Modeling the defect as a finite square well potential with height 5eV and width 200pm, calculate the ground state energy ?
(c) Calculate the total number of bound states with symmetric wavefunction?

6. *A particle of mass m is bound in a double well potential shown in the figure. Itsenergy eigenstate $\psi(x)$ has energy eigenvalue $E = V$ o (where V_o is the energy of the plateau in the middle of the potential well). It is known that $\psi(x) = C$ (C is a constant) in the plateau region.



(a) Obtain $\psi(x)$ for the regions -2 L $<$ x $<$ $-$L and $L < x < 2L$ and the relation between the wavenumber ' k ' and L.

(b) Determine ' C ' in terms of L.

(c) Assume that the bound particle is an electron and $L = 1A$. Calculate the 2 lowest values of V_o (in eV) for which such a solution exists.

(d) For the smallest allowed k, calculate the expectation values for x, x^2, p and p^2 and show that Heisenberg's Uncertainty Relation is obeyed.