PH-107: Introduction to Quantum Mechanics
Tutorial Sheet 7

* marked problems will be solved in the Wednesday tutorial class

Particle in a Box:

1. * For a particle in a 1-D box of side L, show that the probability of finding the particle between $x = a$ and $x = a + b$ approaches the classical value b/L, if the energy of the particle is very high.

2. Consider a particle confined to a 1-D box. Find the probability that the particle in its ground state will be in the central one-third region of the box.

3. Consider a particle of mass m moving freely between $x = 0$ and $x = a$ inside an infinite square well potential.
(a) Calculate the expectation values $\langle \hat{X} \rangle_n$, $\langle \hat{P} \rangle_n$, $\left\langle \hat{X}^2 \right\rangle_n$, and $\left\langle \hat{P}^2 \right\rangle_n$, and compare them with their classical counterparts.
(b) Calculate the uncertainties product $\Delta x_n \Delta p_n$.
(c) Use the result of (b) to estimate the zero-point energy.

4. Consider a one dimensional infinite square well potential of length L. A particle is in $n = 3$ state of this potential well. Find the probability that this particle will be observed between $x = 0$ and $x = L/6$. Can you guess the answer without solving the integral? Explain how.

5. * Consider a one-dimensional particle which is confined within the region $0 \leq x \leq a$ and whose wave function is $\Psi(x, t) = \sin(\pi x/a) \exp(-i\omega t)$.
(a) Find the potential $V(x)$.
(b) Calculate the probability of finding the particle in the interval $a/4 \leq x \leq 3a/4$.

6. An electron is moving freely inside a one-dimensional infinite potential box with walls at $x = 0$ and $x = a$. If the electron is initially in the ground state ($n = 1$) of the box and if we suddenly quadruple the size of the box (i.e., the right-hand side wall is moved instantaneously from $x = a$ to $x = 4a$), calculate the probability of finding the electron in:
(a) the ground state of the new box and
(b) the first excited state of the new box.

7. Solve the time independent Schrodinger equation for a particle in a 1-D box, taking the origin at the centre of the box and the ends at $\pm L/2$, where L is the length of the box.

8. * Consider a particle of mass m in an infinite potential well extending from $x = 0$ to $x = L$. Wave function of the particle is given by

$$\psi(x) = A\left[\sin\left(\frac{\pi x}{L}\right) + \sin\left(\frac{2\pi x}{L}\right)\right]$$

where A is the normalization constant

(a) Calculate A

(b) Calculate the expectation values of x and x^2 and hence the uncertainty Δx.

(c) Calculate the expectation values of p and p^2 and hence the uncertaintyΔp.

(d) What is the probability of finding the particle in the first excited state, if an energy measurement is made?

(given, $\int_0^L x\cos\left(\frac{n\pi x}{L}\right)dx = 0$, $\int_0^L x^2\cos\left(\frac{n\pi x}{L}\right)dx = 0$, for all n)

9. Suppose we have 10,000 rigid boxes of same length L from $x = 0$ to $x = L$. Each box contains one particle of mass m. All these particles are in the ground state.

(a) If a measurement of position of the particle is made in all the boxes at the same time, in how many of them, the particle is expected to be found between $x = 0$ and $L/4$?

(b) In a particular box, the particle was found to be between $x = 0$ and $L/4$. Another measurement of the position of the particle is carried out in this box immediately after the first measurement. What is the probability that the particle is again found between $x = 0$ and $L/4$?

10. * An electron is bound in an infinite potential well extending from $x = 0$ to $x = L$. At time $t = 0$, its normalized wave function is given by

$$\psi(x,0) = \frac{2}{\sqrt{L}}\sin\left(\frac{3\pi x}{2L}\right)\cos\left(\frac{\pi x}{2L}\right)$$

(a) Calculate $\psi(x,t)$ at a later time t.

(b) Calculate the probability of finding the electron between $x = L/4$ and $x = L/2$ at time t.

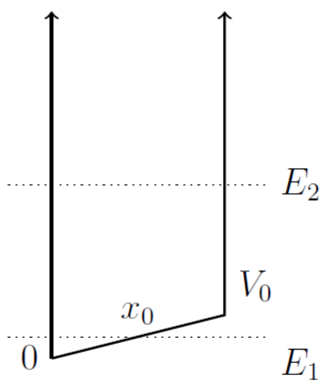
11. A speck of dust ($m = 1$ μg) is trapped to roll inside a tube of length $L = 1.0$ μm. The tube is capped at both ends and the motion of the speck is considered to be along the length of the tube.

(a) Modeling this as a 1-D infinite square well, determine the value of the quantum number n if the speck has an energy of 1 μJ.

(b) What is the probability of finding this speck within 0.1 μm of the center of the tube ($0.45 < x < 0.55$).

(c) How much energy is needed to excite this speck to an energy level next to 1 μJ? Compare this excitation energy with the thermal energy at room temperature ($T =$ 300 K).

12. Consider a particle bound inside an infinite well whose floor is sloping (variation is small) as shown in the figure. Without solving the Schrodinger equation (provide proper justification for your answers),



(a) sketch a plausible wave function when the energy is E_1, assuming that it has no nodes.

(b) Sketch the wave function with 5 nodes when the energy is E_2.