PH-107: Introduction to Quantum Mechanics

Tutorial Sheet 6

* marked problems will be solved in the Wednesday tutorial class

Free particle

1. *Show that

$$\psi(x) = A\ \sin(kx) + B\ \cos(kx)$$

and

$$\psi(x) = Ce^{ikx} + De^{-ikx}$$

are equivalent solutions of TISE of a free particle. A, B, C and D can be complex numbers.

2. Show that

$$\Psi(x,t) = A\sin(kx - \omega t) + B\cos(kx - \omega t)$$

does not obey the time-dependant Schroedinger's equation for a free particle.

3. The wave function for a particle is given by,

$$\phi(x) = Ae^{ikx} + Be^{-ikx}$$

where A and B are real constants. Show that $\phi(x)^*\phi(x)$ is always a positive quantity.

4. * A free proton has a wave function given by

$$\Psi(x,t) = Ae^{i(5.02*10^{11}x - 8{:}00*10^{15}t)}$$

The coefficient of x is inverse meters, and the coeffcient of t is inverse seconds. Find its momentum and energy.

5. A particle moving in one dimension is in a stationary state whose wave function,

$$\Psi(x) = \begin{cases} 0, & \text{x < -a} \\ A\left(1 + \cos\frac{\pi x}{a}\right), & -a \le x \le a \\ 0, & \text{x > a} \end{cases}$$

where A and a are real constants.
(a) Is this a physically acceptable wave function? Explain.
(b) Find the magnitude of A so that $\psi(x)$ is normalized.
(c) Evaluate Δx and Δp. Verify that $\Delta x \Delta p \geq \hbar/2$.
(d) Find the classically allowed region.

6. * Consider the 1-dimensional wave function of a particle of mass m, given by

$$\psi(x) = A\left(\frac{x}{x_0}\right)^n e^{-\frac{x}{x_0}}$$

where, A, n and x_0 are real constants.

(a)Find the potential $V(x)$ for which $\psi(x)$ is a stationary state (It is known that $V(x) \to 0$ as $x \to \infty$).

(b)What is the energy of the particle in the state $\psi(x)$?