# PH-107: Introduction to Quantum Mechanics
## Tutorial Sheet 4

Only "*" to be solved in the tutorials

## Fourier Transform

1. *If $\phi(k) = A(a - |k|), |k| \le a$, and 0 elsewhere. Where $a$ is a positive parameter and $A$ is a normalization factor to be found.
   (a) Find the Fourier transform for $\phi(k)$
   (b) Calculate the uncertainties $\Delta x$ and $\Delta p$ and check whether they satisfy the uncertainty principle.

2. A wave packet is of the form $f(x) = \cos^2\left(\frac{x}{2}\right)(for - \pi \le x \le \pi)$ and $f(x) = 0$ elsewhere
   (a) Plot $f(x)$ versus $x$.
   (b) Calculate the Fourier transform of $f(x)$, i.e. $g(k) = \int_{-\infty}^{+\infty} f(x)e^{-ikx}dx$ ?
   (c) At what value of $k, |g(k)|$ attains its maximum value?
   (d) Calculate the value(s) of $k$ where the function $g(k)$ has its first zero.
   (e) Considering the first zero(s) of both the functions $f(x)$ and $g(k)$ to define their spreads (i.e. $\Delta x$ and $\Delta k$), calculate the uncertainty product $\Delta x.\Delta k$.

3. *A wave function $\psi(x)$ is defined such that $\psi(x) = \sqrt{2/L}\sin(\pi x/L)$ for $0 \le x \le L$ and $\psi(x) = 0$ otherwise.
   (a) Writing $\psi(x) = \int_{-\infty}^{\infty} a(k)e^{ikx}dk$, find $a(k)$.
   (b) What is the amplitude of the plane wave of wavelength $L$ constituting $\psi(x)$ ?

4. A wave packet is of the form $f(x) = \exp(-\alpha|x|) \cdot \exp(ik_0 x)$ ( for $-\infty \le x \le \infty$) where $\alpha, k_0$ are positive constants.
   (a) Plot $|f(x)|$ versus $x$.
   (b) At what values of $x$ does $|f(x)|$ attain half of its maximum value? Consider the full width at half maxima (FWHM) as a measure of the spread (uncertainty) in $x$, find $\Delta x$
   (c) Calculate the Fourier transform of $f(x)$, i.e. $g(k) = \int_{-\infty}^{+\infty} f(x)e^{ikx}dx$
   (d) Plot $g(k)$ versus $k$.
   (e) Find the values of $k$ at which $g(k)$ attains half its maximum value? Using the same concept of FWHM as in part (b), calculate $\Delta k$ ? Hence calculate the product $\Delta x.\Delta k$
   $\left[\text{ Given : } \int_0^{\infty} e^{-(\alpha-ik)x}dx = \frac{1}{\alpha-ik}\right]$

## Heisenberg Uncertainty Principle

1. Estimate the uncertainty in the position of (a) a neutron moving at $5 \times 10^6$ m s$^{-1}$ and (b) a 50 kg person moving at 2 m s$^{-1}$.

2. A lead nucleus has a radius $7 \times 10^{-15}$ m. Consider a proton bound within nucleus. Using the uncertainty relation $\Delta\text{p}.\Delta\text{r} \geq \hbar/2$, estimate the root mean square speed of the proton, assuming it to be non-relativistic. (You can assume that the average value of $p^2$ is square of the uncertainty in momentum.)

3. For a non-relativistic electron, using the uncertainty relation $\Delta x \Delta p_x = \hbar/2$

   (a) Derive the expression for the minimum kinetic energy of the electron localized in a region of size ' $a$ '.

   (b) If the uncertainty in the location of a particle is equal to its de Broglie wavelength, show that the uncertainty in the measurement of its velocity is same as the particle velocity.

   (c) Using the expression in (b), calculate the uncertainty in the velocity of an electron having energy 0.2keV

   (d) An electron of energy 0.2keV is passed through a circular hole of radius $10^{-6}$ m. What is the uncertainty introduced in the angle of emergence in radians? (Given $\tan\theta \cong \theta$ )

4. A particle of mass $m$ moves in a one-dimensional potential $V(x) = \alpha|x|$ where $\alpha > 0$. Using Heisenberg's uncertainty relation, the minimum total energy of the particle is found to have the form $E_{\text{min}} = AB^{1/3}$. Find $A$ and $B$.