PH-107 (2021): Tutorial Sheet 2

* marked problems will be solved in the Wednesday tutorial class

de Broglie Wavelength:

1. Calculate the wavelength of the matter waves associated with the following:

   (a) A 2000 kg car moving with a speed of 100 km/h.

   (b) A 0.28 kg cricket ball moving with a speed of 40 m/s.

   (c) An electron moving with a speed of 10^7 m/s.

   Compare in each case the result with the respective dimension of the object. In which case will it be possible to observe the wave nature.

2. Show that the Bohr's angular momentum quantization leads to the formation of standing waves by the electrons along the orbital circumference in hydrogen atom.

3. Calculate the de-Broglie wavelength (in nm) for a photon, an electron and a neutron each with an energy of 5 keV (for electron and neutron, the energy refers to non-relativistic kinetic energy). Take $m_e = 500$ keV/c^2 and $m_n = 1000$ MeV/c^2.

4. *Thermal kinetic energy of a hydrogen atom is $\sim k_B T$ and the radius is $\sim r_1$ (= 0.53 Å, radius of the $n = 1$ Bohr orbit). Find the temperature at which its de Broglie wavelength has a value of $2r_1$. Take the mass of the hydrogen atom to be that of a proton.

Interference, Diffraction, YDSE, Davison-Germer experiment :

1. * Buckminsterfullerene are soccer-like balls (called buckyballs) made up of 60 carbon atoms (C_{60}). A double slit experiment is performed using these buckyballs travelling at a velocity of 100 m/sec (slit width = 150 nm and the separation between the slits and the screen, $D = 1.25$ m from the slits).

   (a) Find the de Broglie wavelength of the buckyball.

   (b) Find the distance between the maxima of the resultant interference pattern. Treat the buck balls as point like objects.

   (c) The size of the buckyballs is ∼10 Å. How does the size of the ball compare with the distance between the neighboring maxima of the interference patterns? Is the size of C_{60} likely to affect the visibility of the interference fringes? Find the initial velocity of C_{60} for which the interference fringes start to become difficult to detect?

2. Consider two plane waves, one with a wave vector, $\vec{k}_1 = (2\pi/\lambda)(\vec{x} + \vec{y} + \vec{z})$, and the other with $\vec{k}_2 = (2\pi/\lambda)\vec{z}$. For $\lambda = 500$ nm, (a) find the resultant wave due to the interference of these two waves, (b) calculate the intensity and (c) analyze the interference pattern in the xy-plane, i.e. the condition for maxima and minima.

3. In a Young's double slit experiment, the slit separation is 0.8 mm and the observing plane is 1.6 m away from the two slits.

   (a) Plot the intensity pattern at the observing plane.

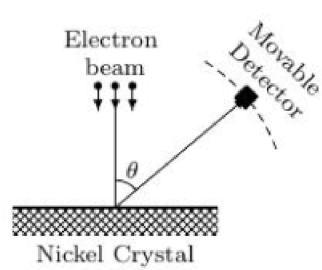
   (b) If the distance between the two consecutive maxima is 5 mm, find the wavelength of the light.

   (c) When one of the slits is covered by a transparent thin film, the central maximum is seen to shift by 2.2 fringes. If the refractive index of the film is 1.4, find the thickness of the film.

   (d) Now the two slits are illuminated by a light containing two wavelengths, 450 nm and 600 nm. What is the least order at which a maximum of one wavelength will fall exactly on a minimum of the other?

4. *In a double-slit experiment with a source of monoenergetic electrons, detectors are placed along a vertical screen parallel to the y-axis to monitor the diffraction pattern of the electrons emitted from the two slits. When only one slit is open, the amplitude of the electrons detected on the screen is $\psi_1(y,t) = A_1 e^{-i(ky-\omega t)}/\sqrt{1+y^2}$, and when only the other is open the amplitude is $\psi_2(y,t) = A_2 e^{-i(ky+\pi y-\omega t)}/\sqrt{1+y^2}$, where A_1 and A_2 are normalization constants. Calculate the intensity detected on the screen when
   (a) both slits are open and a light source is used to determine which of the slits the electron went through and
   (b) both slits are open and no light source is used.
   (c) Plot the intensity registered on the screen as a function of y for cases (a) and (b).

5. *In a Davisson-Germer experiment, electrons having energy of 54eV were bombarded normally over copper crystal. The diffracted beam was recorded using a detector and when the intensity of the diffracted electrons was plotted against the angle with the normal of the surface and the 1st maxima was observed at an angle of $\theta = 35°$.



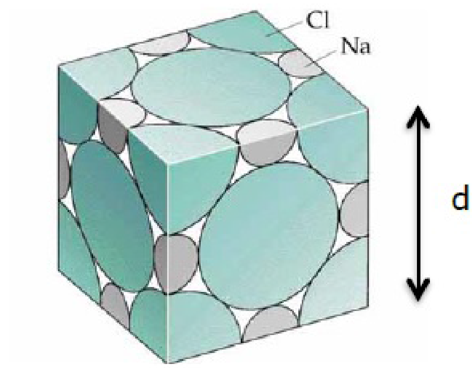
(a) Calculate the spacing between the atoms on the copper surface.

(b) What other angles are possible for a maxima?

(c) If the energy of incident electrons were increased by 3 times. Find the location of first maxima.

(d) How many more intensity peaks (maxima's) will be observed as the angle is further increased?

6. Sodium Chloride (NaCl) crystal is made up of cubes of edge length d, as shown in the figure. Each cube contains a full Na ion at its body center, which is not shown in the figure. In a Davisson-Germer experiment, performed using electrons of kinetic energy 40 eV, the NaCl crystal gives a first order ($n = 1$) diffraction peak at 20.11°.



(a) Compute d

(b) Compute the number of NaCl molecules in the given cube.

(c) Given the density and the molecular weight of NaCl to be 2.17 g/cm^3 and 58.44 g/mol, respectively, compute Avogadro's number.